# TUTORIAL No. 01

## Topic:- MATRICES

Q.1] Show that every square matrix A can be uniquely expressed as $P + iQ$ where $P$ *and* $Q$ are Hermitian matrices .

Q.2] Show that every square matrix can be uniquely expressed as sum of a symmetric matrix and skew symmetric matrix .

Q.3] Show that the matrix $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$. Q.4]Find

$a, b, c$ *if* $A$ *is orthogonal where* $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$

Q.5] Reduce the following matrix to the normal form and hence find its rank .

$$A = \begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

Q.6] Find non –singular matrices P and Q such that PAQ is in normal form. Also find their ranks.

Where $A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$

Q.7] Test for consistency and solve if consistent.

$x - 2y + z - t = 2$ , $x + 2y + 2t = 1$ , $4y - z + 3t = -1$

Q.8] For what values of $\lambda$ the equations

$x + y + z = 1$ , $x + 2y + 4z = \lambda$ , $x + 4y + 10z = \lambda^2$ *have a solution and solve them completely in each case .*

Q.9] Investigate for what values of $\lambda$ *and* $\mu$ the equations

$x + 2y + 3z = 4$ , $x + 3y + 4z = 5$ , $x + 3y + \lambda z = \mu$ have

[i] no solution [ i i] a unique solution [ iii] an infinite number of solution.

Q.10] Solve the following system equations completely

$x - y + 2z + t = 2$ , $3x + 2y + t = 1$ , $4x + y + 2z + 2t = 3$

# TUTORIAL.02

## Topic:-Complex number

1. If arg(z +2i) = $\frac{\pi}{4}$ and arg (z -2i) = $\frac{3\pi}{4}$, find z.
2. If $|z-1|=|z+1|$ then prove that Re z =0.
3. If $\cos\alpha+\cos\beta+\cos\gamma=0$ and $\sin\alpha+\sin\beta+\sin\gamma=0$,
   Prove that $\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\alpha)=0$
4. If $x+\frac{1}{x}=2\cos\theta$ , $y+\frac{1}{y}=2\cos\phi$ , $z+\frac{1}{z}=2\cos\psi$ ,prove that

   $xyz+\frac{1}{xyz}=2\cos(\theta+\phi+\psi)$ and $\frac{x^m}{y^n}+\frac{y^n}{x^m}=2\cos(m\theta-n\phi)$.

5. If $\alpha,\beta$ are the roots of the equation $x^2-\sqrt{3}x+1=0$ ,prove that $\alpha^n+\beta^n=2\cos(n\pi/6)$
   .Hence, deduce that $\alpha^6+\beta^6=-2$ .
6. Use De Moivre's Theorem to show that $\tan 5\theta=\frac{5\tan\theta-10\tan^3\theta+\tan^5\theta}{1-10\tan^2\theta+5\tan^4\theta}$.
7. Show that $\frac{\sin 7\theta}{\sin\theta}=7-56\sin^2\theta+112\sin^3\theta-64\sin^4\theta$
8. Using De Moivre's Theorem prove that, $\cos^6\theta+\sin^6\theta=\frac{1}{8}(3\cos 4\theta+5)$
9. Prove that $\cos^5\theta\sin^3\theta=-\frac{1}{2^7}(\sin 8\theta+2\sin 6\theta-2\sin 4\theta-6\sin 2\theta)$
10. Find the cube roots of unity. If $\omega$ is a complex cube root of unity, prove that
    $\left(1+\omega+\omega^2\right)=0$ *and* $(1-\omega)^6=-27$ .
11. Solve $x^6-i=0$.
12. Find all the values of $(\frac{1}{2}+i\frac{\sqrt{3}}{2})^{\frac{3}{4}}$ and show that their continued product is 1.
13. If $\alpha,\alpha^2,\alpha^3,\alpha^4$ are the roots of $x^5-1=0$, find them and show that
    $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)=5$ .
14. Show that the roots of $(x+1)^7=(x-1)^7$ are given by $\pm\cot\frac{r\pi}{7}$ , $r$ = 1,2,3.

# Tutorial No. 03

## Topic:-Hyperbolic functions and Logarithm of complex numbers

1. If $\tanh x = \frac{2}{3}$, find the value of x and coshx.
2. If $\log \tan x = y$ *prove that* $\cosh ny = \frac{1}{2}\left[\tan^n x + \cot^n x\right]$ and

   $\sinh(n+1)y + \sinh(n-1)y = 2\sinh ny . \operatorname{cosec} 2x.$
3. Prove that $16\sinh^5 x = \sinh 5x - 5\sinh 3x + 10\sinh x$
4. If $\cos(x+iy) = e^{i\alpha}$ *prove that* $(i) \sin\alpha = \pm\sin^2 x = \pm\sinh^2 y$

   $(ii) \cos 2x + \cosh 2y = 2$
5. If $\coth(\alpha + i\frac{\pi}{8}) = x + iy$, *prove that* $x^2 + y^2 + 2y = 1.$
6. Prove that $\sinh^{-1}\tan\theta = \log\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right)$
7. Separate into real and imaginary parts of $\cos^{-1}(e^{i\theta})$
8. Prove that $\log(e^{i\alpha} + e^{i\beta}) = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$
9. If $\sqrt{i}^{\sqrt{i}^{\sqrt{i}\ldots\ldots\infty}} = \alpha + i\beta$ *prove that* $\alpha^2 + \beta^2 = e^{-\pi\beta/2}$
10. Show that for real value of a and b,

$$e^{2ai\cot^{-1}b} = \left[\frac{bi-1}{bi+1}\right]^a$$

# TUTORIAL No. 04

## Topic:- Numerical Method and P.D.

Q.1) Using Regula Falsi Method find the root of the following equation up to four places of decimals

a) $x \log_{10} x - 1.2 = 0$ lying between 2 and 3.

b) $x^3 - 9x + 1 = 0$

Q.1) Using Newton-Raphson Method find the root of the following equation up to four places of decimals

a) $x \sin x + \cos x = 0$ .

b) $\sqrt[3]{15}$

Q.3) Solve the following equations by Gauss –Jacobi's Method

a) $12x + 2y + z = 27$ , $2x + 15y - 3z = 16$ , $2x - 3y + 25z = 26$

b) $5x - y - 2z = 17$ , $2x + 4y = 12$ , $x + 5y + 5z = -1$ *start with* $(2,3,0)$

Q.4 )Solve the following equations by Gauss –Seidel Method

a) $43x + 2y + 3z = 91$ , $3x + 53y + z = 60$ , $2x - 4y + 49z = 49$

b) $28x + 4y - z = 32$ , $2x + 17y + 4z = 35$ , $x + 3y + 10z = 24$

Q.5) If $x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta.$ *Find* $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$

Q.6) If $x = uv,\ y = \frac{u}{v}.$ Prove that J.J'=1

Q.7) Find the maximum and minimum values of $x^3 + xy^2 + 21x - 12x^2 - 2y^2$

# TUTORIAL No. 05

## Topic:- PARTIAL DIFFERENTIATON

Q.1) If $u = \tan^{-1}\left(\frac{x}{y}\right)$, *find the value of* $u_{xx} + u_{yy}$

2) If $u = x^y + y^x$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

3 ) If $\theta = t^n e^{-r^2/4t}$, *find n which will make* $\frac{\partial \theta}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right)$

4) If $u = \sin^{-1}(x - y), x = 3t, y = 4t^3$, *prove that* $\frac{du}{dt} = \frac{3}{\sqrt{1 - t^2}}$

5) If $z = f(u, v)$ *and* $u = x^2 - y^2, v = y^2 - x^2$ *then prove that* $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$

6) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ *then prove that* $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$

7) State and prove Euler's theorem on homogeneous function of two variables, hence find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ *where* $u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$

8) State and prove Euler's theorem on homogeneous function of three variables, hence find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ *where* $u = \frac{x + y + z}{\sqrt{x} + \sqrt{y} + \sqrt{z}}$

9) If $3u = \log\left(\frac{x^3 + y^3}{x^2 - y^2}\right)$ *prove that* $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-1}{3}$

10) If $u = \tan\left(\frac{xy + yz + zx}{x^2 + y^2 + z^2}\right) + \sin\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)$ *then prove that*

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{1}{2}\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)\cos\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)$$

# TUTORIAL.06

## Topic:- Successive Differentiation ,Expansion and Limit

1) Find n[th] derivative of following functions.

a) $e^{ax}\sin(bx+c)$ b) $\frac{x^2}{(x-1)(2x+3)}$ c) $cos^2 x sin^3 x$

2) If $y = x\,log\,\frac{(x-1)}{(x+1)}$, prove that $y_n = (-1)^n (n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$

3) If $y = \frac{x}{x^2+a^2}$, prove that $y_n = (-1)^n . n!\, a^{-n-1} \sin^{n+1}\theta \cos(n+1)\theta$

Where $\theta = \tan^{-1}\left(\frac{a}{x}\right)$

5) If $y = \mathrm{a}\cos\log x + \mathrm{b}\sin\log x$, prove that

$x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$

6) If $y = e^{m\cos^{-1}x}$, prove that

$(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - (n^2 - m^2)y_n = 0$

7) If $y^{1/m} + y^{-1/m} = 2x$, prove that

$(x^2-1)y_{n+2} + (2n+1)x y_{n+1} + (n^2 - m^2)y_n = 0$

8) If $x = \cos\theta\,;\quad \theta = \frac{1}{m}\log y$, prove that

$(1-x^2)y_{n+2} - (2n+1)x y_{n+1} - (n^2 + m^2)y_n = 0$

9) Show that, $\sin(e^x - 1) = x + \frac{x^2}{2} - \frac{5}{24}x^4 + \cdots\ \infty$

10) Prove that, $\log\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right]$ and hence find $\log_e(\frac{11}{9})$

12) Find the values of a,b,c so that $\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x\sin x} = 2$

13) Evaluate $\lim_{x \to 0} \frac{x^2 + 2\cos x - 2}{x\sin x}$