KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 1

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Complex analytic functions

1) Show that $f(z) = ze^z$ is analytic and find its derivative.

2) Find k such that $\frac{1}{2}\log\left(x^2 + y^2\right) + i\tan^{-1}\frac{kx}{y}$ is analytic.

3) Prove that $v = e^{-x}(y\sin y + x\cos y)$ is harmonic, Find its harmonic conjugate and the corresponding analytic function.

4) Find the analytic function such that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ when $f\left(\frac{\pi}{2}\right) = 0$

5) Find an analytic function where, $u + v = e^x(\sin y + \cos y)$

6) Find the orthogonal trajectory of the family of curves $x^2 - y^2 + x = c$

Mapping

7) Find the image of circle | z | = 1 under transformation $w = \frac{5 - 4z}{4z - 2}$

2) Find bilinear transformation under which i, -1, 1of z-plane are mapped to 0, 1, ∞ of w-plane.

3) Find the bilinear transformation which maps points z = 1, i, -1 onto points w = i, 0, -i. Hence find fixed points of the transformation and image of | z | < 1

4) Find the bilinear transformation which maps points z = -1, 1, ∞ onto points w = -i, -1, i.

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Subject In-charge

KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 2

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Vector algebra and Differentiation

1) Prove that, $i\times(\bar{a}\times i)+j\times(\bar{a}\times j)+k\times(\bar{a}\times k)=2\bar{a}$

2) If $\bar{A}=\nabla(xy+yz+zx)$ then find $\nabla\cdot\bar{A}$ *and* $\nabla\times\bar{A}$

3) Find the directional derivative of $\phi=2x^3y-3y^2z$ *at* $(1,2,-1)$ in the direction towards Q(3, -1, 5). Also find the direction and magnitude of maximum directional derivative.

4) Find the angle between two surfaces $x^2+y^2+az^2=6$ *and* $z=4-y^2+bxy$ *at* $P(1,1,2)$

5) Find the values of a, b, c if the directional derivative of $\phi=axy^2+byz+cz^2x^3$ *at* $(1,2,-1)$ has maximum magnitude 64 in the direction parallel to the z-axis.

6) Prove that $\nabla\left[\nabla.\frac{\bar{r}}{r}\right]=-\frac{2}{r^3}\bar{r}$

7) Show that $\bar{F}=(y^2-z^2+3yz-2x)i+(3xz+2xy)j+(3xy-2zx+2z)k$ is both solenoidal and irrotational.

8) Prove that $\bar{F}=(z^2+2x+3y)i+(3x+2y+z)j+(y+2zx)k$ is irrotational and find scalar potential function Φ such that $\bar{F}=\nabla\Phi$, $\Phi(1,1,0)=4$ Hence find the work done by $\bar{F}$ in moving a particle from A(0,1,1) to B(3,0,2)

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KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 3

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Vector Integration

1) Find $\oint [(x^2 + 4)i + (y^2 - 4)j].dr \ where\ C\, is\ x^2 + y^2 = 4$

2) Using Green's Theorem Evaluate $\int_C \overline{F}.d\bar{r}\ \ for\ \ \overline{F} = (x^2 - y^2)i + (x + y)j$, and C is the triangle with vertices (0,0), (1,1), (2,1)

3) Verify Green's Theorem for $\int_C \overline{F}.d\bar{r}\ where\ \ \overline{F} = (x^2 - xy)i + (x^2 - y^2)j$ and C is closed curve bounded by X^2 = 2y and x = y

4) Use Stroke's theorem to evaluate $\int_C \overline{F}.d\bar{r}\ \ for\ \ \overline{F} = yi + zj + xk$ and C is the boundary of the surface $x^2 + y^2 = 1 - z, z > 0$

5) Using Gauss's Divergence theorem evaluate $\iint_S (ax^2 + by^2 + cz^2) dS$ over the sphere $x^2 + y^2 + z^2 = 1$

6) Use Stoke's theorem to evaluate $\int_C \overline{F}.d\bar{r}\ \ where\ \ \overline{F} = -xy\, i + 2yz\ j + y^2 k$ and C is the boundary of half sphere $x^2 + y^2 + z^2 = a^2 , z = 0$

7) Evaluate $\iint_S (yi + xj + z^2k).\overline{ds}$ where S denotes surface bounded by $x^2 + y^2 = a^2\ and$ $z = 0 , z = h$

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KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 4

Class: - S.E. Subject: - Applied Maths- III
Sem: - III

Laplace Transform

Q. 1 Find Laplace transform of the following functions.

1) $t\left(\frac{\sin t}{e^t}\right)^2$

2) $(t\sinh 2t)^2$

3) $\frac{e^{-2t}\sin 2t\cosh t}{t}$

4) $e^{3t}t\,erf\,4\sqrt{t}$

5) $\int_0^t u^{-1}e^{-u}\sin u\,du$

6) $\int_0^t ue^{-3u}\cos^2 2u\,du$

Q. 2 Evaluate the following.

1) $\int_0^\infty \frac{e^{-t}\sin^2 t\,dt}{t}$

2) $\int_0^\infty e^{-8t}\,t\,erf\,9\sqrt{t}\,dt$

Q. 3 Find the Laplace transform of f(t) = |sin pt|, t >= 0 (Periodic)

Q. 4 Express the following functions in terms of Heaviside's unit step function and hence find their Laplace transform

$$f(t)=\begin{cases}\sin t\ , & 0<t\le\pi\\ \sin 2t, & \pi<t\le 2\pi\\ \sin 3t, & t>2\pi\end{cases}$$

Q. 5 Find the Laplace transform of $e^{-t}\sin t\,H(t-\pi)$

Q. 6 Evaluate using Laplace transform $\int_0^\infty e^{-t}(1+3t+t^2)\,H(t-2)\,dt$

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KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 5

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Inverse Laplace Transform

Q. 1 Find Inverse Laplace Transform of following functions.

1) $\frac{s+2}{(s+3)(s+1)^3}$

2) $\frac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$

3) $\frac{s}{(s^4+4a^4)}$

4) $\tan^{-1}\left(\frac{2}{s^2}\right)$

5) $\log\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]$

Q. 2 Find Inverse Laplace Transform of following using Convolution theorem.

1) $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$

2) $\frac{1}{s}\tan^{-1}\left(\frac{s+a}{b}\right)$

Q. 3 Find $\int_0^\infty \cos(tx^2)dx$ and hence , find $\int_0^\infty \cos(x^2)dx$

Q. 4 Find inverse Laplace transform of $\frac{e^{-2s}}{(s^2+8s+25)}$

Q. 5 Solve following differential equations by using Laplace Transform.

$\frac{dy}{dt}+2y+\int_0^t y\,dt=\sin t$ given that y(0) = 1

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KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 6

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Fourier series

Obtain the Fourier series of following functions in the given interval

1) $f(x) = x \sin x \ in \ (0, 2\pi), \ Deduce\, that \ \sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{3}{4}$

2) $f(x) = \begin{cases} x + \frac{\pi}{2}, & -\pi < x < 0 \\ x - \frac{\pi}{2}, & 0 < x < \pi \end{cases}, \ Deduce\, that \ \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + ...$

3) $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4} \ in \ (-\pi, \pi)$

4) $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2 - x), & 1 < x < 2 \end{cases}$

5) Find half range sine series for

$f(x) = lx - x^2 in \ (0, l), \ Deduce\, that \ \frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + ..,$

6) Find half range cosine series for $f(x) = x(\pi - x) \ in \ (0, \pi), \ Deduce\, that \ \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

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KONKAN GYANPEETH COLLEGE OF ENGINEERING,KARJAT.

TUTORIAL NO 7

Class: - S.E. Subject: - Applied Maths- III

Sem: - III

Fourier Transform, Complex form and Bessel Functions

1) Find complex form of Fourier series for $f(x) = \cosh 2x + \sinh 2x \, in \; (-5,5)$

2) Find complex form of Fourier series for $f(x) = e^{ax} \; in \; (-\pi, \pi)$

3) Show that set of functions $\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), ..$ is orthogonal over (0, L)

4) Find the Fourier transform of $f(x) = \begin{cases} (1-x^2), & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right).\cos\frac{x}{2}\,dx$$

5) Find Fourier Integral representation of $f(x) = \begin{cases} e^{ax} & x \le 0,\ a > 0 \\ e^{-ax} & x \ge 0, a > 0 \end{cases}$

6) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}.\sin x$

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