Konkan Gyanpeeth College of Engineering, Karjat

TUTORIAL.01

Applied Mathematics-I (2016-17)

Topic:- MATRICES

Q.1] Show that every square matrix A can be uniquely expressed as $P+iQ$ where P *and* Q are Hermitian matrices .

Q.2] Show that every square matrix can be uniquely expressed as sum of a symmetric matrix and skew symmetric matrix .

Q.3] Show that the matrix $A=\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2=1$. Q.4]Find

a,b,c *if* A *is orthogonal where* $A=\frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$

Q.5] Reduce the following matrix to the normal form and hence find its rank .

$$A=\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

Q.6] Find non –singular matrices P and Q such that PAQ is in normal form. Also find their ranks. Where

$$A=\begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$$

Q.7] Test for consistency and solve if consistent.

$x-2y+z-t=2$, $x+2y+2t=1$, $4y-z+3t=-1$

Q.8] For what values of λ the equations

$x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ *have a solution and solve them completely in each case.*

Q.9] Investigate for what values of λ *and* μ the equations

$x+2y+3z=4$, $x+3y+4z=5$, $x+3y+\lambda z=\mu$ have

[i] no solution [i i] a unique solution [iii] an infinite number of solution.

Q.10] Solve the following system equations completely

$x - y + 2z + t = 2$, $3x + 2y + t = 1$, $4x + y + 2z + 2t = 3$

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TUTORIAL.02

Applied Mathematics-I (2016-17)

Topic:- PARTIAL DIFFERENTIATON

Q.1) If $u = \tan^{-1}\left(\frac{x}{y}\right)$, *find the value of* $u_{xx} + u_{yy}$

2) If $u = x^y + y^x$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

3) If $\theta = t^n e^{-r^2/4t}$, *find n which will make* $\frac{\partial \theta}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right)$

4) If $u = \sin^{-1}(x - y), x = 3t, y = 4t^3$, *prove that* $\frac{du}{dt} = \frac{3}{\sqrt{1 - t^2}}$

5) If $z = f(u, v)$ *and* $u = x^2 - y^2, v = y^2 - x^2$ *then prove that* $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$

6) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$ *then prove that* $\frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{y}\frac{\partial u}{\partial y} + \frac{1}{z}\frac{\partial u}{\partial z} = 0$

7) State and prove Euler's theorem on homogeneous function of two variables, hence find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} \quad where\ u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$$

8) State and prove Euler's theorem on homogeneous function of three variables, hence find the value of

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} \quad where\ u = \frac{x + y + z}{\sqrt{x} + \sqrt{y} + \sqrt{z}}$$

9) If $3u = \log\left(\frac{x^3 + y^3}{x^2 - y^2}\right)$ *prove that* $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{-1}{3}$

10) If $u = \tan\left(\frac{xy + yz + zx}{x^2 + y^2 + z^2}\right) + \sin\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)$ *then prove that*

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{1}{2}\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)\cos\left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)$$